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SUPERDIRECTIVE PROPERTIES OF CLOSED LOOPS OF PARALLEL COPLANAR DIPOLES

Harvard University

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13. ABSTRACT (Maximum 200 words)

This report describes the researches carried out under contract F19628-88-K-0024 from March 28, 1988 through February 28, 1991. After a short review of earlier work, recent progress in the theoretical analysis of resonant loops of dipoles is discussed. In particular, it is pointed out that a way has been found for relating an infinite number of integral equations to a relatively small number. An improvement in the accuracy of the analysis of a resonant circular array of electrically short elements is described and a sequence of twelve basic properties formulated. These relate to the kernel, the conductances, the susceptances, and the horizontal and vertical field patterns. An important conclusion is that any perturbation of the shape of the resonant circular array that results in only a slight change in the currents will significantly change the field pattern and possibly lead to one with superdirective properties. The report concludes with an appendix containing references to the seven published papers generated under this contract.

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SUPERDIRECTIVE PROPERTIES OF CLOSED LOOPS OF PARALLEL COPLANAR DIPOLES

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Report Summary

One of the most important physical phenomena is resonance. Musical instruments, radio, and television depend on tuning strings, pipes, or circuits to resonance at a particular frequency. At resonance the amplitude of vibrations or oscillations reaches a maximum due to the progressive amplification of multiple, in-phase reflections. The purpose of Contract F19628-88-K-0024 has been to explore the properties and possible applications of a remarkable new resonator that combines the characteristics of an antenna array with those of a cavity resonator. It consists of a large number of parallel coplanar dipoles, equally spaced along a closed loop that may be circular, elliptical or serpentine. The single driven element excites waves of electric current in the elements that travel in both directions around the loop. The loop can be tuned to resonance by correctly choosing the length and radius of each element, the distance between them, and their total number. When the loop is resonant, extremely large currents oscillate in all of the elements with phase relations characteristic of any one of a large number of possible modes. These generate a field pattern determined by the phase relations of the currents and the particular shape of the closed loop. Unlike wire and waveguide resonators in which the currents travel the entire length of the circuit with correspondingly large losses, the open resonator involves only transverse currents in very short, widely spaced conductors. Possible applications as a resonant circuit include oscillators and frequency control where its unusual shape and extremely low losses make it useful. Since the conditions of resonance are largely independent of the shape of the closed loop so long as there is no radius of curvature smaller than two wavelengths, an enormous range of possible shapes and associated field patterns is available. These may include particular phase relations, shapes and sizes that lead to extremely high directivity. Such an array would have important practical applications in point-to-point communication and radar. Another application is as a microwave beacon.

The technical problems in the research are theoretical and experimental. They

are related to extremely precise design and construction. The detailed mathematical analysis combined with quadruple-precision numerical calculation has provided designs of which at least one has been verified experimentally (supported by another contract). The general methodology involves a highly precise analytical formulation, its accurate evaluation by computer and, finally, a meticulous verification using an experimental model constructed to high dimensional tolerances. The results obtained to date have been highly encouraging. They show conclusively that this novel open resonator can be designed and built and that it is a device with very unusual properties that can have applications of major importance.

The design of resonant arrays in circular form has been achieved. The problem of further refinement to sharpen the resonances remains to be studied, including the possible need for superconducting elements. Of primary interest for future research is the systematic study of the effects of changes in shape on the field pattern. Can a superdirective array be achieved? The unique properties of single-element excitation, very large resonant currents, purely resistive input impedance, and extremely sharp pencil-like beams already observed for the circular array are encouraging.

I. Review of Work Completed During the Period March 28, 1988 to September 30, 1990

The study of the properties of closed loops of parallel coplanar dipoles began with a systematic review of early work on superdirective endfire arrays, long parasitic arrays of Yagi type, and the resonant properties of circular arrays with only one element driven. This summarizing investigation is contained in the paper "Supergain Antennas and the Yagi and Circular Arrays" by R. W. P. King [1]. It revealed that there is a minimum attenuation design for Yagi-type structures with a hundred or more elements and that this applies not only to linear arrays but to structures of semicircular form provided the radius of curvature is large enough. When two semicircular arrays are combined to form a circular array driven by a single element, traveling waves of current can be excited that propagate around the circle in opposite directions. Clearly, with proper dimensions, the conditions for a resonant system are provided in which the currents successively induced in the elements can build up very large amplitudes.

It was initially thought that elements near an antiresonant length were appropriate and a systematic study of the three-term theory was undertaken as a tool for analyzing circular arrays with elements up to a wavelength long. In the first part of this investigation the properties of the vector potentials of coupled antennas were examined and used to show that great simplification is achieved when the distance between adjacent elements is equal to or greater than the half-length of the elements. This is contained in the paper "Electric Fields and Vector Potentials of Thin Cylindrical Antennas" by R. W. P. King [2]. The complete analysis of circular arrays in which the three-term theory is applied to determine the currents in the elements is contained in the paper "The Large Circular Array with One Element Driven" by R. W. P. King [3]. An important conclusion was that the simpler two-term theory is entirely adequate when the elements are electrically short.

Since the trigonometric functions used in the two- and three-term theories cannot take account of the end effect that characterizes actual currents on tubular elements, a detailed analysis was made to improve the currents near the open ends. This included the derivation of "The Universal Current Distribution near the End of a Tubular Antenna" in a paper by H.-M. Shen and T. T. Wu [4] and "The Combination of the Universal End-Current and the Three-Term Current on a Tubular Antenna" in a paper by H.-M. Shen, R. W. P. King, and T. T. Wu [5].

The application of the two-term theory to 60- and 90-element circular arrays of electrically long and thin elements yielded no sharply resonant arrays. On the other hand, the two-term theory applied to 60- and 90-element circular arrays of electrically short and quite thick tubular elements led to the discovery of a variety of sharply resonant modes with large currents in all elements and radiation patterns consisting of sharp spikes. These modes and the underlying theory are described in detail in the paper "The Resonant Circular Array of Electrically Short Elements" by G. Fikioris, R. W. P. King, and T. T. Wu [6]. The properties of the different resonant modes are related to the resonances of particular phase sequences in the method of symmetrical components used in the analysis. They depend critically on the radius and length of each element and the distance between adjacent elements.

A comparison of the predicted resonance in and the associated field pattern of a 90-element circular array with those observed in an experimental array of 90 brass monopoles on an aluminum ground screen and driven by one element showed quite good agreement. However, the extremely sharp resonances needed for a possible superdirective array clearly require great refinement in both the theory and experiment in order to achieve much higher accuracy. Recent very significant progress in the theoretical treatment is described in the following section.

II. Recent Progress in the Theoretical Analysis of Resonant Loops of Parallel, Coplanar Tubular Dipoles

Some aspects of current work are summarized in the paper "Analytical Studies of Large Closed-Loop Arrays" by G. Fikioris, D. K. Freeman, R. W. P. King, H.-M. Shen, and T. T. Wu [7], which was presented at the recent SPIE OE/LASE'91 Meeting held in Los Angeles, January 20-25, 1991. Most of this material is included in the complete review of recent and current work given in Section III.

An important feature of the improved analysis is that a way has been found for relating an infinite number of integral equations to a relatively small number. This permits great simplification in the analysis of the resonant circular array since the kernel of the finite number of integral equations can be approximated by the much simpler kernel of the infinite linear array. Specifically, the limit of the kernel is the kernel of the infinite linear array; indeed it should be expected that the solution for the circular array with large radius of curvature and many elements should be close to that of the infinite linear array. Physically, the circular array supports waves traveling around it continuously, in both directions and with little attenuation. In the infinite linear array, the so-called *surface waves* [8] travel to infinity. This establishes a definite connection between the infinite linear array—which is governed by an infinite number of integral equations—and the large circular array—which is governed by a finite number of integral equations.

III. Improved Analysis of the Resonant Circular Array of Electrically Short Elements

A. Introduction

In the recent paper by Fikioris, King and Wu [6], it was shown theoretically that a large circular array of electrically short dipoles may possess very narrow resonances. The array consists of N identical, perfectly conducting, parallel, non-staggered elements. Only element 1 is center-driven with a voltage V_1; the other $N-1$ elements are parasitic. The half-lengths and radii of the dipoles are h and a, respectively; it is assumed that $ka \ll 1$ and $a \ll h < \lambda/2$, where $\lambda = 2\pi/k$ is the free-space wavelength. The distance between adjacent elements is $d \gtrsim h$. The existence of narrow resonances has been recently verified experimentally [9]. Resonances occur when N and the radius $\rho = d/[2\sin(\pi/N)]$ of the circle are large.

The N currents satisfy N coupled integral equations and the approximate solution used in [6] was the two-term theory developed by Mack [10], [11]. Thus, the current on each parasitic element is proportional to $(\cos kz - \cos kh)$ and the current on the driven element has an additional term $\sin k(h-|z|)$, where z is the distance from the center of the dipole. In this section it is assumed, for simplicity, that N is even. It follows that the N currents are a superposition of the $N/2+1$ phase-sequence currents $I^{(m)}(z)$, $m = 0, 1, \ldots, N/2$, where $I^{(m)}(z)$ is the current on element 1 when all elements are driven by voltages $V_l^{(m)} = (V_1/N)e^{j2\pi(l-1)m/N}$. The array is at its m^{th} phase-sequence resonance when $I^{(m)}(z)$ becomes large.

Recent investigations [12] show that the kernel of the m^{th} phase-sequence integral equation used in [6] does not take into account the coupling effects with enough accuracy for a resonant circular array. A modified, very simple kernel has been developed [12]; it is obtained by setting $a = 0$ in the self-term of the imaginary part of the kernel used in [6]. This modified kernel is, under suitable conditions, exponentially small in N, suggesting the existence of resonances that are exponentially

narrow. Here use is made of the two-term theory developed by Mack and applied in [6] but with the modified kernel to analyze the circular array at (or near) resonance. Conditions for resonance are developed, the significance of the many parameters involved is examined, and simple formulas for the radiation field are derived. Ways to optimize performance are indicated. The resonant circular array is shown to be a highly directive structure with field patterns that involve many sharp, pencil-like beams.

B. Two-Term Theory and the Modified Kernel

The two-term theory solution [6], [11] for the current on element l is valid subject to the conditions noted in the previous section. The formulas are repeated here, with a notation more convenient for the present purposes:

$$I_l(z) = \begin{cases} \dfrac{j2\pi V_1}{\zeta_0 \Psi_{dR} \cos kh} [\sin k(h - |z|) + T_1(\cos kz - \cos kh)]; & l = 1, \\ \dfrac{j2\pi V_1}{\zeta_0 \Psi_{dR} \cos kh} T_l(\cos kz - \cos kh); & l = 2, 3, \ldots, N, \end{cases} \tag{1}$$

where $\zeta_0 = (\mu_0/\epsilon_0)^{1/2} \simeq 376.73\ \Omega$. The parameter Ψ_{dR} is real and independent of N and d/λ. It is defined in [6], [11]. The coefficients T_l of the shifted-cosine part of the current are complex and depend on all the parameters of the problem. They are obtained by superimposing the phase-sequence coefficients $T^{(m)}$:

$$T_l = \frac{1}{N}\left\{T^{(0)} - (-1)^l T^{(N/2)} + 2\sum_{m=1}^{N/2-1} T^{(m)} \cos\left[\frac{2\pi(l-1)m}{N}\right]\right\}, \tag{2}$$

where

$$\begin{aligned} T^{(m)} &= \frac{P_R^{(m)} + jP_I^{(m)}}{D_R^{(m)} + jD_I^{(m)}} \\ &= \frac{P_{1R} + P_{\Sigma R}^{(m)} + jP_I^{(m)}}{D_{1R} + D_{\Sigma R}^{(m)} + jD_I^{(m)}}; \qquad m = 0, 1, \ldots, N/2. \end{aligned} \tag{3}$$

In (3), the parameters in the numerator and the denominator are all real. The subscript I means that the parameter depends only on the imaginary part of the

modified phase-sequence kernel. Hence, $P_I^{(m)}$ and $D_I^{(m)}$ are independent of the radius a/λ. The subscript $1R$ means that the parameter depends only on the self-term of the real part of the kernel and is therefore independent of N, m and d/λ. The subscript ΣR means that the quantity depends only on the mutual terms of the real part of the kernel and is therefore independent of a/λ. The full formulas for the various parameters are

$$P_{1R} = \int_{-h}^{h} \sin k(h - |z|) K_{1R}(h - z)\, dz, \tag{4}$$

$$P_{\Sigma R}^{(m)} = -\frac{1}{1 - \cos kh} \int_{-h}^{h} \sin k(h - |z|) \times [\cos kh\, K_{\Sigma R}^{(m)}(z) - K_{\Sigma R}^{(m)}(h - z)]\, dz, \tag{5}$$

$$P_I^{(m)} = -\frac{1}{1 - \cos kh} \int_{-h}^{h} \sin k(h - |z|) \times [\cos kh\, K_I^{m)}(z) - K_I^{(m)}(h - z)]\, dz, \tag{6}$$

$$D_{1R} = \frac{1}{1 - \cos kh} \int_{-h}^{h} (\cos kz - \cos kh) \times [\cos kh\, K_{1R}(z) - K_{1R}(h - z)]\, dz, \tag{7}$$

$$D_{\Sigma R}^{(m)} = \frac{1}{1 - \cos kh} \int_{-h}^{h} (\cos kz - \cos kh) \times [\cos kh\, K_{\Sigma R}^{(m)}(z) - K_{\Sigma R}^{(m)}(h - z)]\, dz, \tag{8}$$

$$D_I^{(m)} = \frac{1}{1 - \cos kh} \int_{-h}^{h} (\cos kz - \cos kh) \times [\cos kh\, K_I^{(m)}(z) - K_I^{(m)}(h - z)]\, dz. \tag{9}$$

The various parts of the modified kernel are

$$K_{1R}(z) = \frac{\cos kR_1(z)}{R_1(z)}. \tag{10}$$

$$K_{\Sigma R}^{(m)}(z) = \sum_{l=2}^{N/2+1} \xi_l \cos\left[\frac{2\pi(l-1)m}{N}\right] \frac{\cos kR_l(z)}{R_l(z)}, \tag{11}$$

$$K_I^{(m)}(z) = -\frac{\sin kz}{z} - \sum_{l=2}^{N/2+1} \xi_l \cos\left[\frac{2\pi(l-1)m}{N}\right] \frac{\sin kR_l(z)}{R_l(z)}, \tag{12}$$

where

$$\xi_l = \begin{cases} 1, & l = N/2+1, \\ 2, & \text{otherwise}, \end{cases} \tag{13}$$

and

$$R_l(z) = (z^2 + b_l^2)^{1/2}; \qquad b_l = \begin{cases} a, & l = 1, \\ \dfrac{d\sin[(l-1)\pi/N]}{\sin(\pi/N)}, & l \neq 1. \end{cases} \tag{14}$$

Note that the radius a does not appear in (12). Finally, the relation between $T^{(m)}$ and the phase-sequence admittances is

$$Y^{(m)} = G^{(m)} + jB^{(m)} = \frac{j2\pi}{\zeta_0 \Psi_{dR} \cos kh} [\sin kh + T^{(m)}(1 - \cos kh)], \tag{15}$$

and the self- and mutual conductances $G_{1,l}$ (susceptances $B_{1,l}$) are determined only by the phase-sequence conductances $G^{(m)}$ (susceptances $B^{(m)}$):

$$\begin{aligned} Y_{1,l} &= G_{1,l} + jB_{1,l} \\ &= \frac{1}{N}\left\{Y^{(0)} - (-1)^l Y^{(N/2)} + 2\sum_{m=1}^{N/2-1} Y^{(m)} \cos\left[\frac{2\pi(l-1)m}{N}\right]\right\}. \end{aligned} \tag{16}$$

The modified kernel (10)-(12) has been evaluated asymptotically subject to the conditions $N \to \infty$ and d/λ fixed (to be more precise, the arc-length spacing $k\rho/N$ is fixed) with $d/\lambda < m/N \leq \frac{1}{2}$ [12]. $K_{\Sigma R}^{(m)}(z)$ is well approximated by the kernel of the infinite linear array (replace b_l by ld in (11) and let $N \to \infty$ while keeping m/N fixed). It is therefore roughly independent of N for large N and fixed m/N. The imaginary part $K_I^{(m)}(z)$ must be approximated more carefully because the imaginary part of the kernel of the infinite linear array is exactly zero. The asymptotic formula for $K_I^{(m)}(z)$ is

$$\frac{K_I^{(m)}(z)}{k} \sim -\frac{1}{4\pi^{1/2}} \frac{1}{N^{1/2}} \frac{1}{[(m/N)^2 - (d/\lambda)^2]^{3/4}} \exp[-2N(m/N)g(x_m)]$$

$$+ \{\text{same with } m \to N - m\}, \tag{17}$$

where

$$x_m = \frac{d/\lambda}{m/N}; \qquad 0 < x < 1 \quad \text{and} \quad g(x) = \cosh^{-1}\left(\frac{1}{x}\right) - (1 - x^2)^{1/2}. \tag{18}$$

The approximation is better when z/λ is small and when d/λ is not very close to m/N. These are the cases of interest. When $m \ll N/2$, only the first term needs to be kept; in the extreme case of $m = N/2$, the second term simply contributes a factor of 2. The function $g(x)$ appearing in the exponential is positive and decreasing, with $g(0) = \infty$. The following properties of $K_I^{(m)}(z)$ are noted:

Property (1): $K_I^{(m)}(z)$ is approximately independent of z when $d/\lambda < m/N$.

Property (2): $K_I^{(m)}(z)$ is exponentially small in N for fixed d/λ and fixed m/N with $d/\lambda < m/N$.

Property (3): $K_I^{(m)}(z)$ is a rapidly decreasing function of d/λ when N and m/N are fixed with $d/\lambda < m/N$.

With the use of Property (1), approximations for $P_I^{(m)}$ and $D_I^{(m)}$ are obtained when h/λ is not too large. Thus,

$$D_I^{(m)} \sim -2(\sin kh - kh\cos kh)\,\frac{K_I^{(m)}(0)}{k}; \qquad d/\lambda < m/N, \tag{19}$$

$$P_I^{(m)} \sim 2(1 - \cos kh)\,\frac{K_I^{(m)}(0)}{k}; \qquad d/\lambda < m/N, \tag{20}$$

so that $D_I^{(m)}$ and $P_I^{(m)}$ are slowly varying functions of kh that also possess the Properties (2) and (3) above.

C. Phase-Sequence Resonances

Throughout this section d/λ is the variable and a/λ, h/λ and N are fixed, just as in the figures in [6]. The physical picture is similar when the frequency is varied and the geometrical parameters a, h, and d are fixed.

A typical plot of $D_R^{(m)} = D_{1R} + D_{\Sigma R}^{(m)}$ as a function of d/λ is given in Fig. 1. It is seen that $D_R^{(m)}$ is a quantity of order 1 that has two zeros in the range $0 < d/\lambda \leq 0.5$. The array is at its m^{th} phase-sequence resonance when $D_R^{(m)}$ is exactly zero. In Fig. 1, the smaller root is not in the region of validity of the two-term theory since $d < h$. Since the position of the resonance is determined only by the real part of the kernel, a particular m/N phase-sequence resonance will occur at roughly the same value of d/λ for all large N. Hence, it is meaningful to examine resonant currents, etc., as N becomes larger, while keeping m/N and d/λ fixed. Denote by $\delta_{m/N}$ the position of the larger root, so that $D_R^{(m)} = 0$ when $d/\lambda = \delta_{m/N}$. It is seen from (3) and (15) that, at the m^{th} phase-sequence resonance,

$$T_{\text{res}}^{(m)} = \frac{P_I^{(m)}}{D_I^{(m)}} - j\,\frac{P_R^{(m)}}{D_I^{(m)}}, \tag{21}$$

$$G_{\text{res}}^{(m)} = \frac{2\pi}{\zeta_0 \Psi_{dR} \cos kh}\,\frac{P_R^{(m)}}{D_I^{(m)}}, \tag{22}$$

$$B_{\text{res}}^{(m)} = \frac{2\pi}{\zeta_0 \Psi_{dR} \cos kh}\left[\sin kh + (1 - \cos kh)\,\frac{P_I^{(m)}}{D_I^{(m)}}\right], \tag{23}$$

where $P_R^{(m)}$, $P_I^{(m)}$ and $D_I^{(m)}$ are evaluated at $d/\lambda = \delta_{m/N}$.

The quantity $P_R^{(m)}$ is of order 1. Because of Properties (2) and (3) and equations (19), (20) and (16), if $\delta_{m/N} < m/N$, it is seen that

Property (4): At the m^{th} phase-sequence resonance, the phase-sequence conductance $G_{\text{res}}^{(m)}$ is exponentially large in N. The self- and mutual conductances $G_{1,l}$ around the array are also exponentially large and they vary around the array according to

$$G_{1,l} \propto G_{\text{res}}^{(m)} \cos\left[\frac{2\pi(l-1)m}{N}\right]; \quad l = 1, 2, \ldots, N. \tag{24}$$

Property (5): $G_{\text{res}}^{(m)}$ and the $G_{1,l}$'s will be much larger when the resonant spacing $d/\lambda = \delta_{m/N}$ occurs at a smaller value.

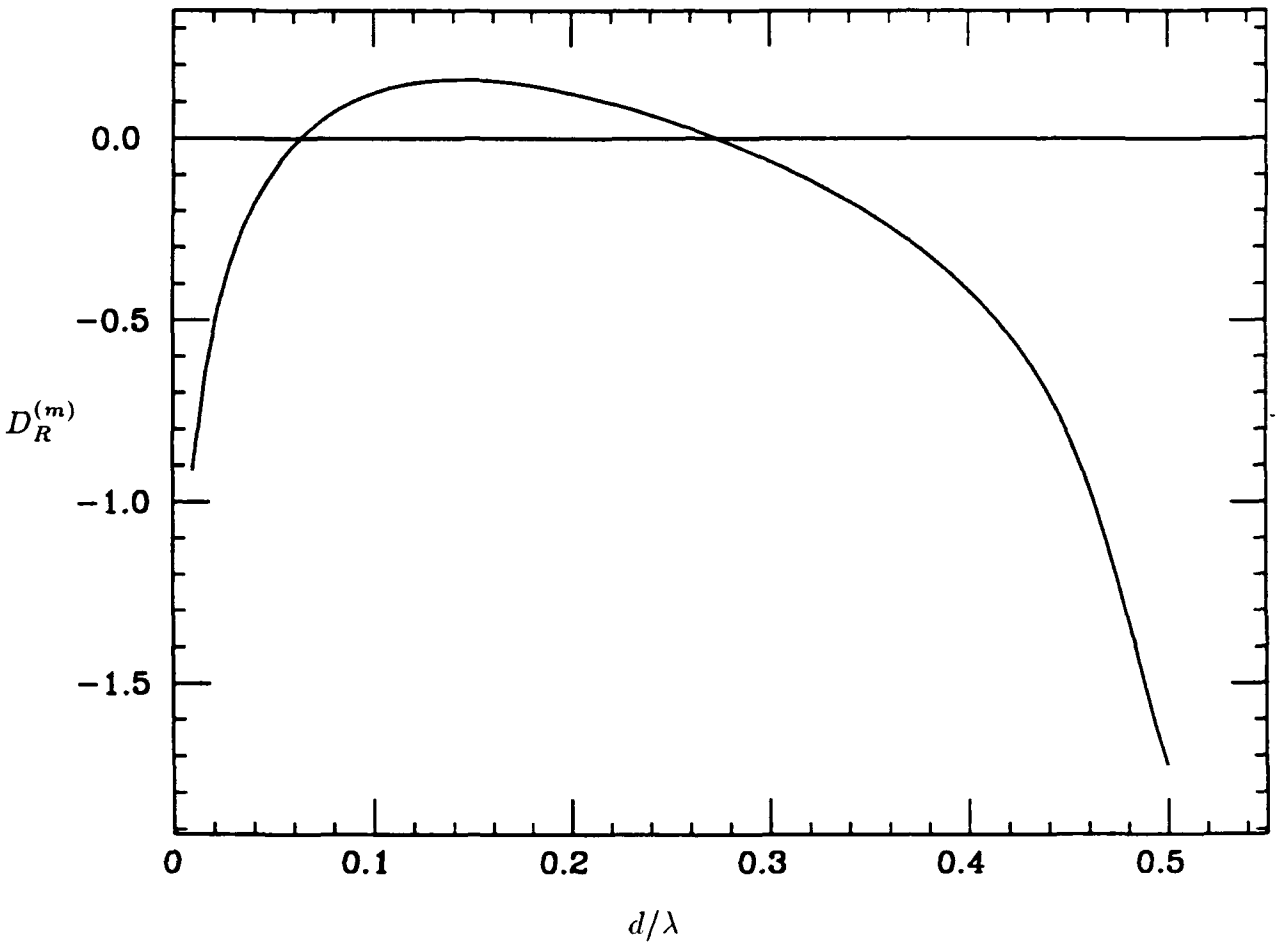


Fig. 1. $D_R^{(m)}$ as a function of d/λ; $N = 90$, $m = 45$, $h/\lambda = 0.2$ and $a/\lambda = 0.05$.

On the other hand, $B_{\text{res}}^{(m)}$ and the $B_{1,l}$'s are not large when the array is exactly at resonance. The magnitudes $|Y_{1,l}|$ of the self- and mutual admittances are determined entirely by the conductances, so that $Y_{1,l} \propto |\cos[2\pi(l-1)m/N]|$. The phases $\text{Arg}(Y_{1,l})$ can be either 0° or 180°; in the special case when $\cos[2\pi(l-1)m/N] = 0$, the current on element l is very small compared to that on all other elements. These observations are generally consistent with the figures in [6]. The differences are due to the fact that in [6] the resonances are not as narrow.

The parameter $D_R^{(m)} = D_{1R} + D_{\Sigma R}^{(m)}$ depends on a/λ only through D_{1R} and on d/λ only through $D_{\Sigma R}^{(m)}$. D_{1R} is zero when the elements are self-resonant. By plotting D_{1R} for various values of h/λ, it is seen that D_{1R} is a decreasing function of a/λ, at least when $0.001 < a/\lambda < 0.07$ and $a/\lambda \ll h/\lambda < 0.22$. (It can be shown, in fact, following a procedure similar to that in Appendix II of [3], that the variation with a is linear in the quantity $\Omega = 2\ln(2h/a)$, but the approximation is poor when h/a becomes small.) Hence, making the dipoles electrically thicker will result in decreasing the amplitude of a curve like that in Fig. 1, thereby shifting the resonant spacing $\delta_{m/N}$ to a smaller value of d/λ. The resonant currents will therefore become much larger. When the elements are electrically very thin, the array can have no narrow resonances at all, because the resonant root occurs at a value $d/\lambda > m/N$. This is the case in Fig. 1 of [6].

The effect of changing the length h/λ is much more involved, since both D_{1R} and $D_{\Sigma R}^{(m)}$ depend on h/λ in a complicated way. However, extensive numerical calculations show that the position of the root $\delta_{m/N}$ decreases when h/λ increases, at least when a/λ and h/λ are in the above-mentioned ranges. Table 1 shows the resonant spacings $\delta_{m/N}$, the values of $K_I^{(m)}(0)$ evaluated at $d/\lambda = \delta_{m/N}$ and the self-conductance $G_{1,1}$, for 90-element arrays at their $m = N/2$ phase-sequence resonance as a/λ and h/λ vary.

Table 1

Resonant Spacings $d/\lambda = \delta_{m/N}$;
Values of the Imaginary Part of the Kernel $K_I^{(m)}(0)/k$ at $z = 0$, $d/\lambda = \delta_{m/N}$;
and the Driving-Point Conductance $G_{1,1}$ at Resonance.
Number of Elements $N = 90$; Phase Sequence $m = N/2 = 45$.
Roots $\delta_{m/N}$ are Sought in the Interval $h/\lambda < d/\lambda < \frac{1}{2} = m/N$.

h/λ	$a/\lambda = 0.01$	$a/\lambda = 0.03$	$a/\lambda = 0.05$
0.14	no root	no root	$\delta_{m/N} = 0.479$ $\frac{K_I^{(m)}(0)}{k} = -0.25$ $G_{1,1} = 16.6$ mA/V
0.16	no root	$\delta_{m/N} = 0.480$ $\frac{K_I^{(m)}(0)}{k} = -0.18$ $G_{1,1} = 11.8$ mA/V	$\delta_{m/N} = 0.439$ $\frac{K_I^{(m)}(0)}{k} = -4.8 \times 10^{-3}$ $G_{1,1} = 132$ mA/V
0.18	$\delta_{m/N} = 0.494$ $\frac{K_I^{(m)}(0)}{k} = -0.47$ $G_{1,1} = 5.2$ mA/V	$\delta_{m/N} = 0.431$ $\frac{K_I^{(m)}(0)}{k} = -2.1 \times 10^{-3}$ $G_{1,1} = 196$ mA/V	$\delta_{m/N} = 0.370$ $\frac{K_I^{(m)}(0)}{k} = -3.9 \times 10^{-7}$ $G_{1,1} = 9.6 \times 10^5$ mA/V
0.20	$\delta_{m/N} = 0.437$ $\frac{K_I^{(m)}(0)}{k} = -4.1 \times 10^{-3}$ $G_{1,1} = 90$ mA/V	$\delta_{m/N} = 0.336$ $\frac{K_I^{(m)}(0)}{k} = -7.0 \times 10^{-10}$ $G_{1,1} = 4.2 \times 10^8$ mA/V	$\delta_{m/N} = 0.273$ $\frac{K_I^{(m)}(0)}{k} = -2.3 \times 10^{-16}$ $G_{1,1} = 1.2 \times 10^{15}$ mA/V

In Table 1, the imaginary part of the kernel was evaluated from (12) using quadruple precision and the self-conductance was calculated using the exact formulas (1)-(16), with $D_R^{(45)}$ set equal to zero. The extremely large currents when the elements are long and thick require perfectly conducting elements; they could be realized in practice only with superconducting elements. The conclusion is that

Property (6): If a specific phase-sequence resonance is desired, making the dipoles electrically longer or thicker will require an electrically smaller circle and will result in much higher resonant currents around the array, at least when a/λ and h/λ are in the ranges $0.001 < a/\lambda < 0.07$ and $a/\lambda \ll h/\lambda < 0.22$.

Of course, the resonant currents have a complicated explicit dependence on h/λ and a/λ as well as this implicit dependence through $d/\lambda = \delta_{m/N}$, but the latter is dominant.

D. Behavior near a Phase-Sequence Resonance

Consider again that a/λ, h/λ and N are fixed and that d/λ is varied, but very close to a resonant spacing $\delta_{m/N}$, so that the array is very close to its m^{th} resonance. The function $D_R^{(m)}$ is usually a quantity of order 1, but near resonance it is of the order of magnitude of the exponentially small quantity $D_I^{(m)}$; it is the controlling quantity in (3). It is a good approximation to assume that $P_R^{(m)}$, $P_I^{(m)}$ and $D_I^{(m)}$ are constant and that $D_R^{(m)}$ varies linearly with d/λ: $D_R^{(m)} \propto d/\lambda - \delta_{m/N}$. Using $P_R^{(m)} \gg P_I^{(m)}$, it is seen from (3) that $\mathrm{Re}\{T^{(m)}\}$ has extrema when $D_R^{(m)} \simeq \pm D_I^{(m)}$, with the corresponding values:

$$\mathrm{Re}\{T^{(m)}\} = \pm \frac{P_R^{(m)}}{2D_I^{(m)}} = \mp \frac{1}{2}\,\mathrm{Im}\{T_{res}^{(m)}\} = \mp\,\mathrm{Im}\{T^{(m)}\}. \tag{25}$$

From the relations (15) between $T^{(m)}$, $G^{(m)}$ and $B^{(m)}$, and (16) between $B_{1,l}$ and

$B^{(m)}$, it is seen that

Property (7): $B^{(m)}$ and the $B_{1,l}$'s are very rapidly varying near a narrow resonance. When the spacing d/λ is such that $G^{(m)}$ has decreased to half its maximum value, $B^{(m)}$ is equal to $G^{(m)}$: $B^{(m)} = \pm G^{(m)} = \pm \frac{1}{2} G_{\text{res}}^{(m)}$. Hence, $B^{(m)}$ and the $B_{1,l}$'s have a zero very close to resonance. The $B_{1,l}$'s vary around the array as $\cos[2\pi(l-1)m/N]$.

It is therefore possible to design an array near resonance with a purely resistive driving-point impedance, but this property will be extremely sensitive to slight changes in the parameters. It is interesting to note that the circular array is similar in behavior to a single center-driven dipole [13]. In both cases, when the driving-point conductance is maximum as a function of f, the susceptance is nearly zero. When the element radius becomes larger, the Q rises.

The Q of the resonant array was defined in [6]. It may be estimated from the curve of $G_{1,1}$ as a function of d/λ as

$$Q_r \simeq \frac{\delta_{m/N}}{(d/\lambda)_2 - (d/\lambda)_1}. \tag{26}$$

(The actual definition involves the frequency.) In (26), the $(d/\lambda)_2$ and $(d/\lambda)_1$ are the spacings at which the power is reduced to one-half the maximum at constant voltage, i.e., when $D_R^{(m)} = \pm D_I^{(m)}$. Approximating $D_R^{(m)}$ as a linear function, viz., $D_R^{(m)} \simeq \alpha(d/\lambda - \delta_{m/N})$, and using $P_R^{(m)} \gg D_I^{(m)}$, it is seen that

$$Q_r \simeq \frac{|\alpha| P_R^{(m)}}{(D_I^{(m)})^2}. \tag{27}$$

It should be noted that it is very difficult numerically to calculate quantities near resonance. $D_R^{(m)}$ is the difference between two nearly equal, complicated integrals of order 1 and must be evaluated with very high precision. Formula (20) provides a simple way to estimate the Q. A formula similar to (20) for the bandwidth may also be obtained.

E. Radiation Field at or near Resonance

With the two-term theory currents, and with the center of the spherical coordinates (r, θ, ϕ) placed at the center of the array, the radiation field is given by [11]:

$$\begin{aligned}\mathbf{E}(r,\theta,\phi) &= \hat{\theta} E_\theta \\ &= -\frac{V_1}{\Psi_{dR}\cos kh}\,\frac{e^{-jkr}}{r} \\ &\quad \times \left\{ F(\theta) e^{jk\rho \sin\theta\cos(\phi-\phi_1)} + G(\theta)\sum_{l=1}^{N} T_l e^{jk\rho\sin\theta\cos(\phi-\phi_l)} \right\}, \end{aligned} \tag{28}$$

where $\phi_l = 2\pi(l-1)/N$ is the location of element l, and

$$F(\theta) = \frac{\cos(kh\cos\theta) - \cos kh}{\sin\theta}, \tag{29}$$

$$G(\theta) = \frac{\sin kh\cos(kh\cos\theta)\cos\theta - \cos kh\sin(kh\cos\theta)}{\sin\theta\cos\theta}. \tag{30}$$

In (28), the first term represents radiation from the sine current of the driven element; the second term is radiation from the shifted-cosine currents of all elements. At or near a narrow resonance, we have $T_l \simeq T_1\cos[2\pi(l-1)m/N]$. It will be seen that the first term in (28) may be neglected, so that it is convenient to define the array factor for the m^{th} phase-sequence resonance as

$$A^{(m)}(\theta,\phi) = \frac{1}{T_1}\sum_{l=1}^{N} T_l e^{jk\rho\sin\theta\cos(\phi-\phi_l)} \tag{31a}$$

$$= \sum_{l=1}^{N} \cos\left[\frac{2\pi(l-1)m}{N}\right] e^{jk\rho\sin\theta\cos(\phi-\phi_l)}, \tag{31b}$$

so that the radiation field is approximately

$$E_\theta = -\frac{V_1}{\Psi_{dR}\cos kh}\,\frac{e^{-jkr}}{r}\,G(\theta)T_1 A^{(m)}(\theta,\phi). \tag{32}$$

Thus, the array factor is the radiation field due to a circular array of isotropic radiators with the m^{th} phase-sequence resonance currents around the array, element 1 having unit current. The sum in (31b) may be written exactly as follows; see [14] for a detailed derivation:

$$\begin{aligned} A^{(m)}(\theta,\phi) &= Nj^m J_m[N(d/\lambda)\sin\theta]\cos(m\phi) \\ &\quad + N\sum_{p=1}^{\infty} j^{Np-m} J_{Np-m}[N(d/\lambda)\sin\theta]\cos[(Np-m)\phi] \\ &\quad + N\sum_{p=1}^{\infty} j^{Np+m} J_{Np+m}[N(d/\lambda)\sin\theta]\cos[(Np+m)\phi]. \end{aligned} \tag{33}$$

Because of the condition $d/\lambda < m/N \leq \frac{1}{2}$, the arguments of the Bessel functions are always smaller than the orders. When N is large, only two terms in (33) are significant, viz.,

$$\begin{aligned} A^{(m)}(\theta,\phi) &\sim Nj^m J_m[N(d/\lambda)\sin\theta]\cos(m\phi) \\ &\quad + \{\text{same with } m \to N-m\}. \end{aligned} \tag{34}$$

As with the imaginary part of the kernel, the first term is adequate when $m \ll N/2$ and the second term is equal to the first one when $m = N/2$. Assuming for simplicity that $m \ll N/2$ and using the asymptotic formula for the Bessel functions, one obtains

$$\begin{aligned} A^{(m)}(\theta,\phi) &\sim j^m \frac{1}{(2\pi m/N)^{1/2}} \frac{1}{[1-(x_m\sin\theta)^2]^{1/4}} \\ &\quad \times \exp[-N(m/N)g(x_m\sin\theta)]\cos(m\phi), \end{aligned} \tag{35}$$

where x_m and $g(x)$ are defined in (18). Hence, the array factor is an exponentially small quantity and, in fact, it shares Properties (2) and (3) of the imaginary part of the kernel. This is due to cancellation. Also, it has a zero of order m at $\theta = 0$. From (35), the field formula (32), and the expressions (19)–(21) for $T_1 \simeq (2/N)T_{\text{res}}^{(m)}$, we

see that:

Property (8): The magnitude of the radiation field at any fixed point in space is exponentially large in N.

That is, the resonant currents are large enough to overcome the cancellation effects. Radiation from the sine current of element 1 is negligible.

Property (9): The horizontal field pattern ($\theta = \pi/2$) consists of $2m$ spikes.

Property (10): The vertical field pattern is very narrow, with a maximum at $\theta = \pi/2$.

An exponential largeness of the field should be expected since, for lossless elements, integration of $|E_\theta|^2$ over a large sphere should give the total radiated power $\frac{1}{2}G_{1,1}|V|^2$, which is exponentially large. The narrowness of the vertical beam can be estimated by neglecting variations of the field in (32) due to the slowly varying $G(\theta)$ and defining vertical directivity as the maximum of the array factor divided by its mean value, viz.,

$$D_V = \frac{|A^{(m)}(\pi/2, \phi)|}{\frac{1}{2}\int_0^\pi |A^{(m)}(\theta, \phi)| \sin\theta \, d\theta}. \tag{36}$$

Subject to the approximation (34), the integral can be carried out analytically. Thus,

$$D_V = \frac{2J_m[N(d/\lambda)]}{\pi J_{(m-1)/2}[\frac{1}{2}N(d/\lambda)]J_{(m+1)/2}[\frac{1}{2}N(d/\lambda)]} \sim \frac{2J_m[N(d/\lambda)]}{\pi J^2_{m/2}[\frac{1}{2}N(d/\lambda)]}, \tag{37}$$

where the last approximation is valid for large m. With the asymptotic expression for the Bessel functions,

$$D_V \sim (2N/\pi)^{1/2}[(m/N)^2 - (d/\lambda)^2]^{1/4}. \tag{38}$$

Hence,

Property (11): For a specific phase-sequence resonance m/N, making N larger will result in a narrower vertical field pattern, and in more spikes in the horizontal plane.

Property (12): For fixed N and for a specific phase-sequence resonance m, making the dipoles thicker or longer will result in a smaller resonant spacing $\delta_{m/N}$, a much narrower resonance, and a slightly more directive vertical field pattern.

The directivity may therefore be made arbitrarily large by making N large (although the increase is slow, as the square root of N). The field strength at any point in space increases exponentially. The input impedance may be a pure resistance. However, the physical dimensions of the array increase (linearly with N) and the bandwidth decreases exponentially.

The array factor's smallness has an interesting consequence. For resonant noncircular arrays, an array factor $A(\theta, \phi)$ may be defined exactly as in (31a). This array factor will depend on the array's geometry and the relative current distribution around the array. It will be a sum of N terms of order 1, each term depending on the location of element l and its relative current (admittance). If the noncircular array is thought of as a perturbation of some corresponding circular array, then it is logical to assume that the current distribution around the array will not be significantly affected. Hence, each term in the sum for $A(\theta, \phi)$ will be close to each term in the sum for the circular array. However, any very small quantity that can be written as the sum of terms of order 1 is extremely sensitive to perturbations of these terms. Therefore, the array factor (field pattern) for the noncircular array will not be close to that of the circular array. The conclusion is that any perturbation of the resonant circular array's shape resulting in only a slight change in the current distribution around the array will significantly change the field pattern. This means that a wide variety of field patterns may be obtained by resonant noncircular arrays, perhaps even a superdirective pattern. However, one must be very careful in dealing with noncircular arrays. The N linear algebraic equations for the currents, to which the N coupled integral equations reduce by the two-term theory approximations, must be solved with high accuracy. The currents must be known precisely in order to obtain any reasonable estimate of the far-field pattern.

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